

TRICKY PROBLEMS

1. **Find only solution in R for $(4x^2+6x+4)4y^2-12y +25)=28$?**

Sol: It is an equation involving two variables and one equation which cannot be solved directly. Some strategy must be required to get the solution. With the help of inequality we are able to solve the problem.

$$\text{Let } A = (4x^2 + 6x + 4)(4y^2 - 12y + 25)$$

$$= \left(\left(2x + \frac{3}{2} \right)^2 + \frac{7}{4} \right) \left((2y - 3)^2 + 16 \right)$$

$$\text{If } (x, y) \neq \left(\frac{-3}{4}, \frac{3}{2} \right) \text{ then } A > 16 \left(\frac{7}{4} \right) = 28$$

$$A = 28 \text{ is possible only when } (x, y) = \left(\frac{-3}{4}, \frac{3}{2} \right)$$

2. **Let x be a real number such that $x^3 + 4x = 8$. Determine The value of $x^7 + 64x^2$?**

Sol: Given equation is third degree equation; it cannot be solved so easily. So getting 'x' value is difficult. If getting 'x' value is difficult then how do we evaluate value of higher degree expression? By reducing higher powers of 'x' to lesser powers of 'x' we slowly evaluate value of $x^7 + 64x^2$

$$x^3 + 4x = 8$$

$$x^3 = 8 - 4x$$

$$N = x^7 + 64x^2 = x(x^3)^2 + 64x \dots (1)$$

Putting in the value for x^3 in equation (1) we get

$$\begin{aligned} x(8 - 4x)^2 + 64x^2 &= (64x - 64x^2 + 16x^3) + 64x^2 \\ &= 16(x^3 + 4x) = 16 \cdot 8 = \mathbf{128}. \end{aligned}$$

3. **Find all real solutions to these equation**

$$\frac{w^2+1}{w^2} \times \frac{x^2+1}{x^2} \times \frac{y^2+1}{y^2} \times \frac{z^2+1}{z^2} = 1?$$

Sol: There is only one equation and more variables.

But each term of LHS is greater than 1, where as RHS is equal to one. Hence there is no real solution.